## A practical guide to

## OFF-AXIS PARABOLOID ALIGNMENT PROCEDURE



## 1. GEOMETRY

The geometry of an off-axis section of a paraboloid is shown schematically in Figure 1a below:

Figure 1a: Definition of tangential and sagittal planes in relation to the geometry of an off-axis paraboloid.


The tangential plane is defined as passing through the mechanical centre of the off-axis section and the optical axis as shown. The sagittal plane also passes through the mechanical centre of the mirror and the focus of the parabola but is perpendicular to the tangential plane.

Figure 1 b below, shows a section through the mirror in the tangential plane with the important parameters shown.

Figure 1b: Important
parameters that characterise
an OAP
$f=$ focal length
$\mathrm{h}=$ off-axis distance
$\mathrm{g}=$ apparent focal length
a = off-axis angle


The defining relationships between these parameters are:
$h=2 \cdot f \cdot \tan \left(\frac{a}{2}\right)$
$f=g \cdot \cos ^{2}\left(\frac{a}{2}\right)$
Where $f$ : is the true focal length (i.e. distance between vertex and focus)
g: is apparent focal length (i.e. distance between mechanical centre and focus)
a: is the off-axis angle
Equation (1) gives the relationship between the off-axis distance, $h$, and the off-axis angle, a.
Equation (2) gives the relationship between the true focal length, $f$, and apparent focal length $g$, at a given off-axis angle.

## 2. INITIAL ALIGNMENT

On completion of the manufacturing process the parameters of the off-axis parabolic mirror are measured and a fiducial mark inscribed on the edge to indicate the direction of the vertex (see Figure 2). Thus the arrow points toward the optical axis.

Initial alignment is normally carried out using these measured parameters to set the mirror in approximate adjustment.
It should be noted that this procedure is subject to the usual mechanical imprecisions of both measurement and setting. Thus the adjustment at this stage may not be sufficiently accurate to produce the optimum performance of the mirror.

## 3. TYPES OF ERRORS

The best way to eliminate the remaining alignment error is by use of an interferometer as shown in Figure 2. This can be the Optical Surfaces Ltd Scatterplate, or LUPI interferometers, or a Fizeau type interferometer such as those made by Müller Wedel, Wyko, Zygo etc. A plane mirror is used to return the collimated light from the parabola back to the common focus of the interferometer and parabola.

Figure 2:


In this type of testing, the mirror is used twice, or in a "double pass". Any errors seen are therefore double the actual wavefront errors of the mirror under test, and four times the surface errors.

The wavefront error measured by the interferometer may be interpreted to indicate the alignment errors. These errors are of three main types:
i) Focus and tilt
ii) Incorrect setting of off-axis distance
iii) Incorrect setting in tangential plane

## i). Focus and Tilt

These are easily understood and are indicated by the presence of circular or elliptical fringes in the case of focus. Straight fringes or a shift of the centre of the fringe pattern indicate tilt errors.

As a guide to the sensitivity to a focal shift the equation below may be used.

$$
W=\Delta s=\frac{d^{2}}{8 f^{2}} \Delta f
$$

Where $f$ is the focal length
$\Delta f$ is the focal shift
$d$ is the diameter of the mirror
$\Delta s$ is the resultant change of sagittal depth of the reference sphere $W$ is the wavefront error

## ii). Incorrectly set Off-Axis Distance

If the off-axis distance is set incorrectly (equivalent to an incorrect setting of the off-axis angle) then a comatic error may be observed. Since we are dealing with an off-axis section of an axially symmetric parabolic mirror this error will appear as astigmatism (at least for small misalignments).

Light from the tangential and sagittal planes will thus focus at different points on the optical axis.
Typical interferograms are shown in figure 3. Figure 3a shows an example at best focus. Figures 3b and 3c show the effect of defocus. Note the characteristic elliptical fringes with the major/minor axis of the ellipse parallel to the tangential plane.

Again, as an indication of the magnitude of the wavefront error due to coma, the following equation may be used:

$$
W=\frac{\left(r_{0}^{3}-r_{i}^{3}\right) \Delta h}{4 f}
$$

Where, $r_{0}$ is the distance between the optical axis and the outside edge of the mirror (i.e. half the diameter of the parent parabola)
$r_{\mathrm{i}} \quad$ is the distance between the optical axis and the inside edge of the mirror.
$f \quad$ is the true focal length
$\Delta h \quad$ is the error of setting of the axial displacement
$W \quad$ is the wavefront error

## Figure 3:

Interferograms due to incorrect setting of off-axis distance.
a) At best focus
b \& c) With defocus


## iii). Incorrect Alignment of the Mirror in Rotation

Errors of this type arise due to a rotational misalignment of the mirror about its mechanical centre. This has the effect of shifting the "true optical axis" of the mirror out of the tangential plane as shown in Figure 4.

As above, this error is comatic in nature. However, since this is an out of plane error, the sense of the coma is inclined at $45^{\circ}$ to the tangential plane.

Figure 4:
Misalignment due to rotation of sector about its mechanical centre.

The fiducial marks on the mirror correspond to the dashed line.


Figure 5 illustrates this type of error. Figure 5a shows an example at best focus and the subsequent ones show the effect of defocus and tilt. Again note the characteristic elliptical fringes when defocused. Now the major/minor axis is inclined at $45^{\circ}$

To calculate the wavefront error due to this rotational misalignment the following equation may be used:
$W=\frac{d^{3} t}{32 f^{3}}$
Where, $f$ is the focal length
$d$ is the diameter of the mirror
$t$ is the distance between the "true" focus and the tangential plane

Figure 5:
Interferograms due to rotational error about the mechanical centre of the mirror.
a) At best focus
b \& c) With tilt
d \& e) With defocus


## iv). Mixed errors

Figure 6, shows the resultant interferograms when a system contains a mixture of all error types.
With defocus, the fringes still show an elliptical nature but now the major/minor axis is no longer in any particular direction - its direction is dependent on the exact mixture off the error types. Such interferograms are typical of a mirror which has been crudely aligned.

Figure 6:
Interferograms due to a combination of alignment errors.

a \& b) Errors of off-axis distance and rotational at best focus
c \& d) As above with tilt
e, f, g, h) Off-axis distance, rotation and defocus

## 4. FINAL ALIGNMENT - Step by step instructions

## Step 1:

In order to optimise the alignment it is best to eliminate each type of error in turn. Before doing so it is generally easiest to deliberately defocus the system (if it is not so already). It should then be possible to obtain the fringes of the type shown in Figures 7a, 8a (The direction of the major axis of the elliptical fringes is not important and may indeed be $\approx 90^{\circ}$ to those shown - the direction is a function of the magnitude and sense of the errors).

## Step 2:

Starting with rotational errors, rotate the mirror about its mechanical centre. This will alter the position angle of the axis of the elliptical fringes, and the eccentricity of these fringes. The correct direction of rotation may easily be determined by inspection of the fringes, since the correct sense will reduce the eccentricity of the ellipse. The effect of this is shown in Figures 7a-d, and 8a-d. Note that depending on the nature of the combination of error types, the major axis of the fringes may rotate clockwise or anticlockwise.

## Step 3:

Continue this rotation until the major axis of the fringes is either parallel or perpendicular to the tangential plane (Figures 7d, 8d).

If it is inconvenient to rotate the mirror, the same effect may be achieved in the following way - alter the "height" of the interferometer (movement in the sagittal plane) and bring the image back to the interferometer by tilting the mirror or reference flat appropriately.

The system now contains no errors of mirror rotation.

## Step 4:

Proceed now to eliminate errors due to off-axis setting. Either rotate the mirror about its vertical axis through the mechanical centre of the mirror and perpendicular to the tangential plane or laterally displace the mirror. Again the correct direction is easily determined by inspection of the fringes - the correct direction will reduce the eccentricity. When the fringes are circular (Figures $7 e$ and $8 e$ ) the system is fully aligned and it is now merely necessary to focus the system correctly.

## Step 5:

If the initial alignment has left the mirror with a large amount of misalignment it may be necessary to repeat the above procedure more than once - reducing the errors on each successive iteration.

After optimizing the mirror alignment some errors may remain. These will probably relate to residual errors in the mirror surface and should be similar to those apparent in the interferogram supplied with the mirror.

As previously noted, the fringes seen are four times as sensitive as the surface error measurement by which the mirror was specified. If unaccountable errors are present, this may be due to the test flat, distortion of either mirror caused by the mount or errors in the interferometer.

Please contact us if further assistance is required.

## Figure 7:

Typical interferograms seen during alignment
a) Initial setting
b, c, \& d) Result of rotating mirror about its mechanical centre
e) Circular fringes resulting from the final adjustment of off-axis distance

a


b


e

Figure 8:
As Figure 7 but with a different combination of error types. Compare the direction of rotation of the major axis of the elliptical fringes.


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