High power piezoelectric axial shockwave generation

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1 Introduction

Mechanical impacting is involved in a lot of technical processes, e.g. for demolition of concrete by chiselling, for structure borne sound analysis, for impact-echo analysis of extended structures, for characterization of material properties at high strain rates or indentation hardness tests.

For a precise understanding of this paper a redefinition of the used "shock-related" terms is needed:

When two rigid bodies (bars or rods) are colliding, then for a short time they are interacting by inducing a rapid change of mechanical stress within their materials. It is assumed, that this interaction is purely elastic and no plastic deformation occurs. This elastic stress propagates into the bodies with their particular speed of sound and this phenomenon is called "stress impulse". In English literature often the inconsistent expression "shock wave" is used to describe this situation.
When it is done here, "shockwave" and "stress impulse" are synonyms.

The above ideal situation of shock propagation has to be distinguished from the technically used effects, when an impact is to be created by the indentation of a tool's cutting edge (e.g. from a chisel or drill) into the surface of a rigid counterpart. This is generally a non-linear interaction, where the related constitutive law depends self-evidently on the properties of the target material.
The extreme situations are plasticity deformations without material removal on one side and chipping/crushing of highly brittle material like ceramics on the other side.

A detailed understanding of shock wave generation is the key for increasing the efficiency of impact processes.

For generating an impact, in most cases, a hard mass body is accelerated during a starting phase (e.g. a hammer head), colliding then with a hard counterpart (e.g. chisel). By this interaction of µs duration, impulse and energy are transferred between the bodies. The details of this process are defined by the acousto-elastic properties of the impact partners.

The starting and contact phases of such a classical impact scenario are hard to be reproduced exactly. Common setups of precision impact measurement do not allow high repetition rates.
Further the special feature of a µs-precise timing of the impact event cannot be done by classical setups.

These restrictions can be overcome by piezomechanical technology providing an adaptive impact generation principle, where the impact parameters and the timing can be controlled by the electrical driving conditions.

This paper describes the shockwave generation by axially active piezo-stacks featuring following essentials.

No pre-impact acceleration phase:

- The impact partners are brought into mechanical contact before the impact generation.
- The complete system is at rest before igniting the impact process.

- Shockwave parameters like energy content, pulse duration, acceleration rates are preset by the electrical pulse parameters.

- Shocks can be generated with high repetition rates and high reproducibility (1kHz within a burst).

- µs precise time of the shock event

- Impact generation at non accessible sites

- Miniaturization (e.g. for shock sensor calibration)

- Impact generation under exotic driving conditions (e.g. cryogenic temperatures)

- High acceleration rates (up to 500.000 m/s^2)

- High forces (up to several 10 kN depending on the shock generator design).

The cooperation of HILTI AG and PIEZOMECHANIK GmbH aimed for the investigation of high power levels by using big-sized high load piezo-stacks.

The elastic collision 2

2.1 Empirical approach

A physical shockwave is defined as the generation of local deformation/disturbance within an elastic medium (e.g. a steel bar) by a rapid process (e.g. collision with a hammer head). The resulting stress distribution is then propagating through the medium with the velocity of sound (approx. 5 km/s in steel). Physical impulse and energy are carried by this shock wave.

Related to the stress pulse propagation is a displacement of the material by the compression act. The velocity of this displacement is the so called "particle velocity". This velocity is smaller by several orders of magnitude compared to the speed of sound (range of several m/s).
Particle velocity and involved mass displacement are defining the related physical impulse.

The details of shock propagation depend strongly on the nature and shape of the collision partners. The response of interacting spheres differs from the collision of bars (due to the excitation of internal degrees of freedom).
Collision experiments (e.g. for material characterization with high strain rates) are often carried out by using metal bars, because this situation can be handled straight forward by the mechanical theory of thin bars (see chapter 9).
The dynamic deformation/strain of such metal bars is detected by strain gages, the particle velocity by LDAs (LaserDopplerAnemometer) monitoring the bar's surface.

Shockwave propagation becomes more complex when passing either into another kind of material or if the cross section of the bar is changing. The pulse is split into reflected and transmitted parts. The quantitative analysis of this situation allows the characterization of the high dynamic elastic properties of material probes under very high strain rates as carried out by split Hopkinson bar experiments (Figure 2).

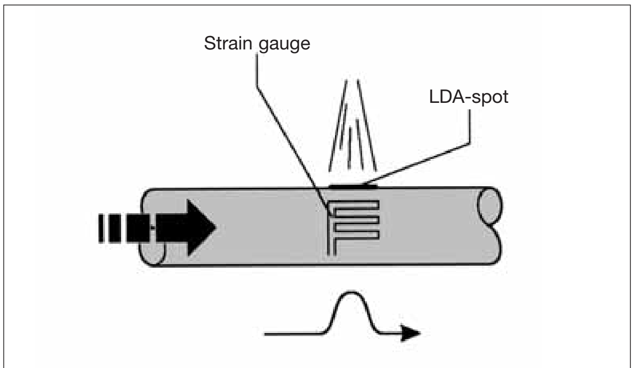

Figure 1: *Schematic of a shock wave propagation within a solid bar, detected by strain gage and LDA measurements.*

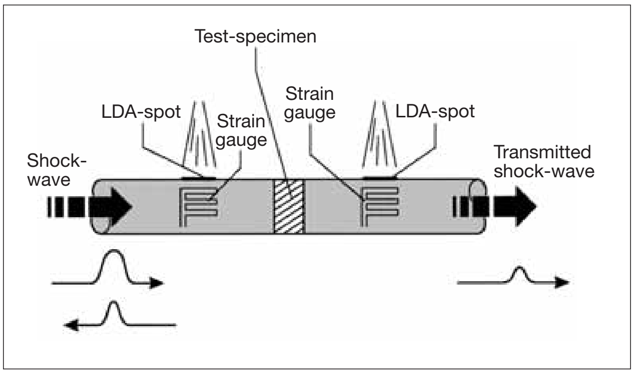

Figure 2: *Hopkinson bar experiment. The exciting and transmitted shockwaves are measured by strain gages and LDA.*

2.2 Collision mechanics

In practice, the following basic shockwave situations are usually discussed

A Free moving bar end: no collision partner.
A propagating shockwave will be completely reflected back into the bar.
Bar's end is moving with maximum particle velocity.
The acceleration rate of bar's end is doubled by the reversed motion.

B Bar's end blocked and cannot move:
At bar's end the particle velocity is zero mechanical stress/compression is maximal.

C Acoustical matching:
Describes the shockwave transfer at an interface e.g. between two metal bars.
The shockwave passes the matched interface without losses in energy and impulse.
For optimizing the acoustical match, the materials and cross sections of the interacting bars are essential.

D Acoustical mismatch:
complementary situation to c.
The incoming mechanical pulse is partially reflected and transmitted at the contact point.
Energy is transferred incompletely (see figure 2).
The shockwave reflection at a seismic mass is the extreme case of an acoustical mismatch:
The complete shockwave energy is reflected (see figure 5).

Shock wave generation by piezo-stacks

3

3.1 The piezo-stack: an active bar

A piezo-stack can be described as a solid bar of PZT-ceramics. When this PZT bar is electrically charged sufficiently fast, the internal stress jumps instantaneously to a high level:
The initial pressure is the blocking pressure, causing now an accelerated expansion of the PZT-stack: a shock is created. By coupling the PZT rod to another solid body, the shock impulse can be transferred and a shock wave is propagating.
In this terminology a PZT-stack is an "active bar", generating inherently mechanical shock pulses by electrical pulse excitation.
Because the active bar is at rest, two pulses are generated propagating in opposite direction to balance the total impulse to zero.
This kind of pulse generation is called to be "super-elastic", because the kinetic energy of the system after the shock generation is higher than before the shock event.
This situation is similar to the use of an explosive.

3.2 Symmetrical shock generator

The above mentioned basic symmetry of the shock-generation within a resting active bar can be used for a symmetrical arrangement with two opposing shock outputs of identical impulse content.
This allows elegant designs for calibration experiments.

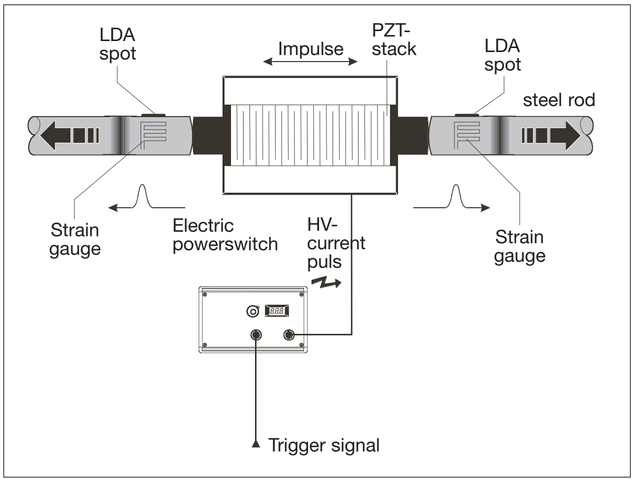

Figure 3: *Symmetrical piezo-shock-generator with two-sided shock propagation. The shock parameters are detected by strain gages and LDAs.*

Figure 4b: *High load piezo-stack with 35 mm diameter, with shock proof wiring.*

Design of a symmetrical piezo-shock-generator
The shockwave is generated within the piezo-bar and transferred to both ram elements (steel, titanium, brass etc.). Acoustical matching for maximum energy extraction is done by adoption of the cross sections of PZT-stack and the metal rams.

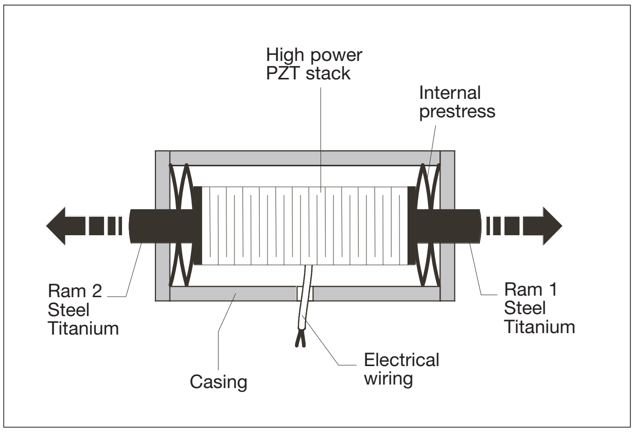

Figure 4a: *Schematic of a symmetrical piezo-shock-generator.*

Figure 4c: *Symmetrical shock generator: shock output via the rams (left and right).*

3.3 Single end piezo-shock-generator

The above mentioned symmetrical generator setup can be modified towards a single ram element with a nearly doubled energy and impulse output. This is done by applying a bigger mass for supporting the PZT-stack at one side (seismic mass). The impulse will be reflected there and directed towards the other output. A double pulse is created by containing nearly double the pulse energy (by elongating the total pulse output duration). The compensating impulse is transferred to the seismic mass as recoil (similar to a rifle).

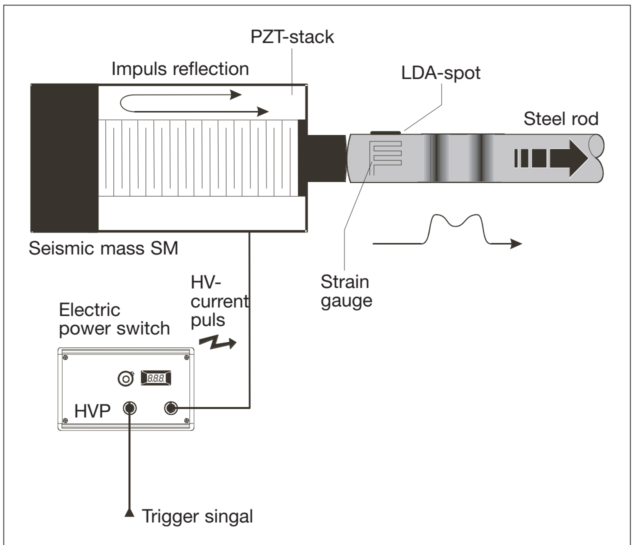

Figure 5: *Schematic of a single end piezo-shock-generated system by application of a seismic mass. The interaction of the travelling pulses results in an elongated double pulse output.*

Design of a single end piezo-shock-generator

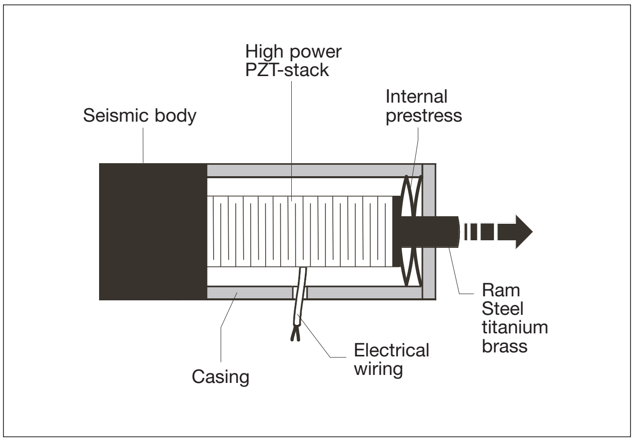

Figure 6a: *Schematic of a single output piezo-shock-generator.*

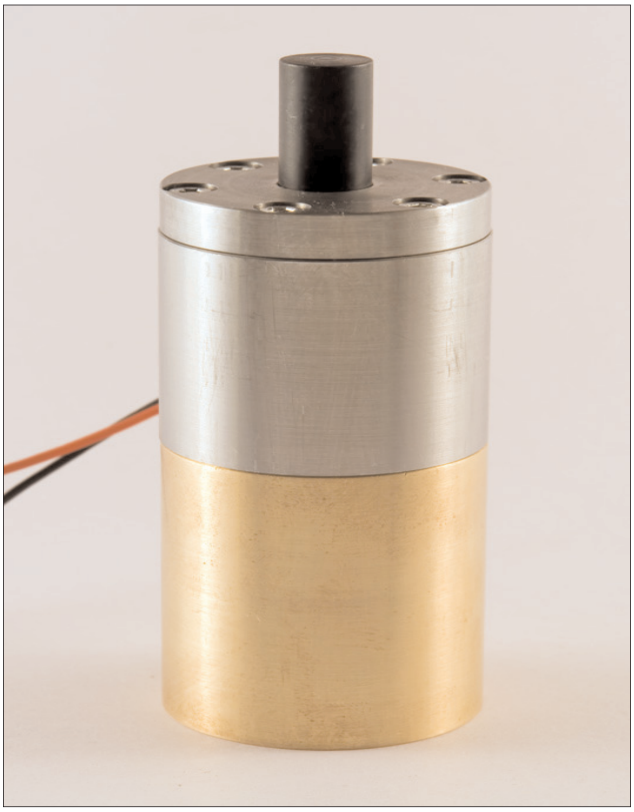

Figure 6b: *Single ouput piezo-shock-generator: brass part: seismic countermass.*

Characterizing piezo-stack based shockwave generators

4

The piezo-mechanical performance parameters of a piezo-shock-generator can be derived straight forward from the characterization of piezo-stack actuators.

- Free ram stroke:
  Depends on PZT-stacks length, the applied electrical field strength and type of the used PZT ceramics. Strokes are ranging from µm to hundreds of µm.
  An efficient energy transfer must be completed within this stroke range limit.
  Piezo-shock-generators need therefore hard collision partners.

- Internal starting force:
  Generated force level within the PZT-bar upon complete electrical charge transfer.
  When the electrical rise-time is significantly shorter than the shock generator's mechanical rise-time, the initial force corresponds to an actuator's blocking force. It depends mainly on stack's cross section, applied electrical field strength and type of used PZT material.
  This starting force accelerates then the masses of the active PZT rod.

- Pulse-width:
  = shock propagation time through the active PZT rod.
  It depends therefore on speed of sound in the PZT stack and stack's length.
  By pulse reflection at a seismic base, the resulting pulse length is doubled.

- Particle velocity:
  It corresponds to the maximum achievable shift speed of a piezo-actuator.
  Order of magnitude: several m/s.

- Mechanical energy content:
  It depends on volume or mass of the PZT-stack, the applied electrical field strength and the properties of the PZT ceramics.

- Impulse content P of the mechanical shock:
  P = moved mass within the shock front multiplied by the related particle velocity.
  Order of magnitude: kgm/s.

- Piezo-ceramics:
  High dielectric, high strain PZT materials provide a much higher shock energy density than "low capacitance" actuator PZT materials (up to a factor 2).

Electrical pulse excitation 5

The electrical equivalent circuitry describes a piezoceramic component mainly as a capacitor. By applying a huge charge impulse to this PZT-capacitor (= generator stack), a mechanical shock-wave will be produced. Our experiment aimed for the characterization of large volume, high dielectric stack actuators for a maximum mecha-nical pulse energy output.

A typical arrangement was:
Piezo-generator capacitance: order of magnitude 10 µF
Impedance of charging circuitry during pulse ignition: 1 ohm
Equivalent RC time constant: 10 µs
This time constant is significantly shorter than the transition time of the mechanical pulse.

Applied peak voltage: up to +800 V
Peak current: up to 800 A

It has to be taken into account, that by high field excitation, the effective capacitance is remarkably higher than the above stated small signal value. This effective capacitance must be explicitly derived from the electrical charging parameters.

For shock generation, short-term a huge power level is needed in the order of magnitude 0.5 Megawatts.
It was produced by the following circuitry (see Figure 7).

The HighVoltagePulser HVP consists of a condensator bank of parallel and seriel high voltage capacitors with a total capacitance of typically 500 µRF >> piezo-shock-generator's capacitance.
This storing capacitor is charged up to a level up to +800 V by a standard power supply (not shown in schematic).

By an IGBT power transistor switch (2), the piezo-shock-generator is coupled to this electrical charge reservoir. Due to the very low resistance of this circuitry, a fast charging pulse of the piezo-element occurs.

In terms of maximizing the shock wave energy, two operating modes have been compared, differing in the start conditions for the electrical pulse:

It is well-known from common piezo-actuators, that a much higher mechanical energy output is achieved by the so-called semi-bipolar operation instead of the unipolar mode. "Unipolar mode" means, the electrical pulse is applied to a discharged piezoelement (pre-pulse voltage level 0, voltage step 0V/U_{max} V).

A dramatic increase of the mechanical power output of the shock generator is achieved by a pre-pulse conditioning of the piezo-shock-generator by connecting it to a negative voltage power supply with levels down to -200 V. (switch position 1). To one part, this is simple caused by the wider voltage step applied to the actuator (voltage step -200V/+U_{max}).

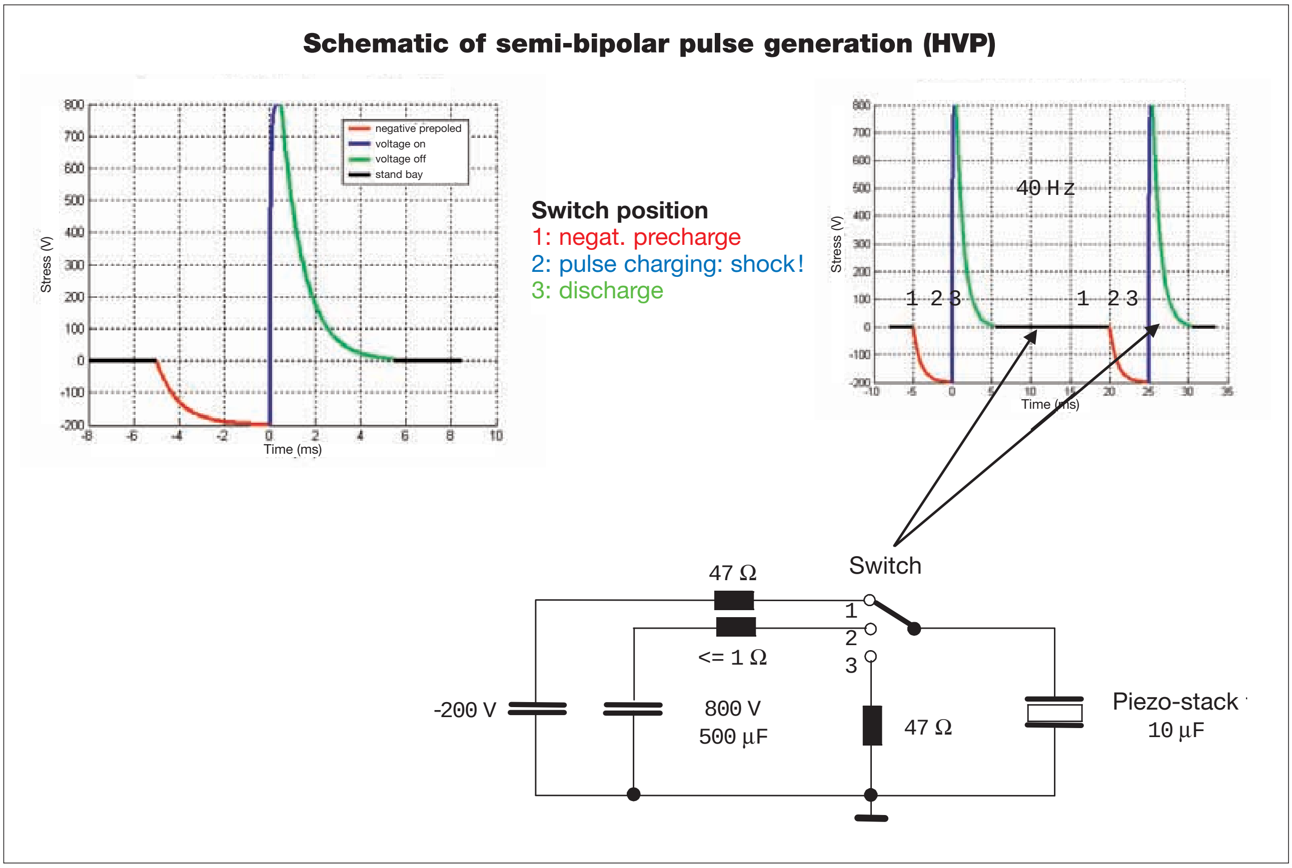

Figure 7: *Schematic of bipolar switching of a power piezo-actuator by a HighVoltagePulser (HVP). The unipolar mode uses 0V instead of the -200V pre-pulse conditioning.*

A further contribution to the power balance is done by the nonlinear poling response by applying an electrical counter field to a high dielectric PZT-ceramic (here by applying the -200V pre-pulse setting).

For resetting the system after the shock wave generation, the piezo-shock-generator is simply discharged by a simple resistor (here 47) via the switch position 3. Burst operation with 100 Hz on the described big size PZT-elements has been successfully carried out.

Experimental studies on piezo-shock-generators

6

The cooperation between HILTI/Schaan/Liechtenstein and PIEZOMECHANIK/Munich/Germany was focusing on high power mechanical shock generation by using high voltage high load piezostacks.

HILTI is the leading manufacturer of highest quality tools for demolition, chiselling based on the electro-pneumatic hammer principle. The high efficiency of these tools is based on the very exact matching of all involved parts for the shock transfer from the hammer head down to the tip of the chisel.
Piezo-shock-generators with their high power levels, high repetition rates and reproducible shock parameters are used for R&D activities to shorten test periods on these components significantly.

6.1 Experimental set up

High voltage high load piezo-stacks with an active diameter 35 were used as the electro-mechanical impulse converter.
Emphasis has been put onto a shock proof design of the stack itself and the applied high current electrodes.
At the very beginning of this project, standard "low dielectric, low capacitance" PZT-actuator ceramics has been ruled out because of its reduced "shock power" efficiency. Consequently all shock generator stacks were based on the highly dielectric high power PZT-material HP from PIEZOMECHANIK.

For an optimum acoustical matching, the piezo-stacks where combined with steel bars of 18 mm diameter.
The shock pulses propagating within the steel bars have been characterized by Laser-Doppler-Anemometry (LDA) determining the particle velocity at bar's surface, when the shock wave passes by. From this particle velocity, the related strain variation has been derived. Together with the well-know elastic properties of steel, the energy and impulse content of the shock front can be evaluated.

6.2 Symmetrical piezo-shock-generator

Figure 8: *Symmetrical pulse generator vertical arrangement. Both sides are coupled to steel bars for two sided impulse extraction. Notice the slits in casing and mounting flanges steel for getting optical access for the LDA-measurements.*

Comparison of the shock profiles within the two steel bars derived from the above mentioned symmetrical arrangement:

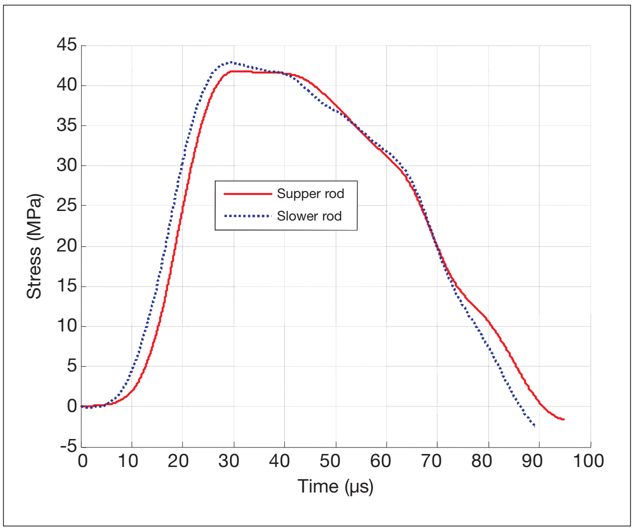

Figure 9: *Shock profiles of a 50 mm active length symmetrical pulse generator, electrical pulse level 500 V.*

Timing is triggered by the transistor signal. The delay of the onset of the mechanical pulse is due to the distance from shock generator to the LDA-measuring spot on the steel bar.

Take notice of the nearly identical shock profiles. The slight offset between the two pulses is due to slight differences of the distances of the LDA-spots from the pulse generator.

6.3 Single output shock generation

The asymmetric shock generator with seismic base uses the same piezo-stacks like the symmetrical design. Figure 10 shows a "head down" arrangement of the shock generator: the contact between the piezo-shock wave generator and the steel bar is thereby preloaded with the weight force of approx. 100 Newtons.

Figure 10: *Test set up using a single output piezo-shock-generator with seismic mass for impulse reflection.*

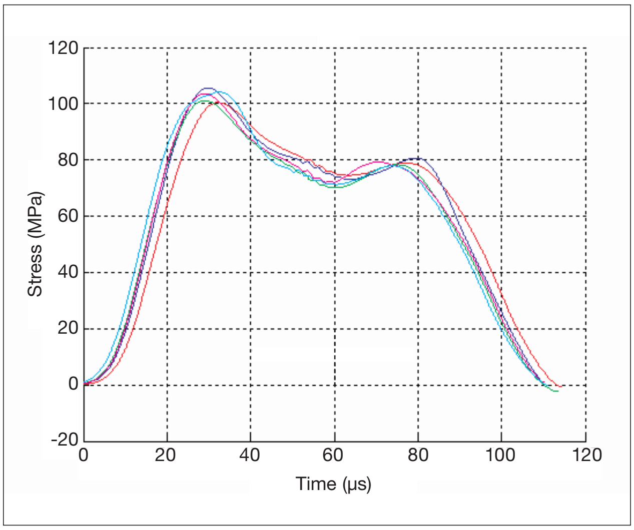

Figure 11: *Mechanical stress/time profile time for a burst of shocks. Active stack length: 120 mm. Shockwave energy: 3.25 J. Take notice of the remarkable coincidence of the mechanical pulses.*

Figure 11 shows the typical single output pulse profile exhibiting a double peak structure of the stress response. This is due to superposition of the primary impulse and the impulse, reflected at the seismic mass. This elongates the total propagation time and leads to longer lasting output pulse.
The energy content is increased nearly to a factor of two compared to a single pulse of a symmetric arrangement.

Piezo-stacks with a length up to 200 mm have been tested with a mechanical shock energy content of more than 4 J (energy measured in the steel bar!). Physical impulse contents up to 2 kgm/s have been verified.
For a "free end" bar experiment acceleration rates up to 500.000 m/s² can be expected at the bar's tip.

7 Discussion

Our experiments verified the specific properties of axially shock wave generation by using piezo-stack-actuators.

A The results fit well into the bar theory of shock propagation as the interaction of axially activated elements.

B High levels of extractable mechanical energy in the range of 7 J/kg piezo-ceramics are achievable, provided a mechanically high efficient material is used (like PIEZOMECHANIK's HP-PZT) and the semi-bipolar operating mode is applied. Then the power balance exceeds that of standard actuator PZT by a factor of two.

C No pre-pulse "external mass acceleration" phase is needed for generating a shock. The whole system is at rest and the shock generator is already in contact with the mechanical counter part immediately before to shock release.

D µs-precise timing of the shock-event by electrical means.

E Variation of shock parameters like amplitude and pulse width by electrical means.

F Excellent reproducibility of shock profiles.

G High repetition rates (in bursts).

8 Outlook

The normalized mechanical parameters of a piezo-stack like strain, blocking pressures, energy density are (nearly) independent of the actual dimensions of the piezo-element.
Therefore piezo-shock-generators can be successfully designed over a wide range of dimensions, ranging from "big block" structures down to miniature shockers with similar electro-mechanical conversion efficiencies.

Shock testing as a kind of DIRAC-pulse application with its very fast rise time, high acceleration level, excellent reproducibility and elevated repetition rates allow mechanical testing/calibration of components like acceleration/crash sensors. New approaches for quality inspection, mode analysis, structure borne sound evaluation are under investigation as an alternative to harmonic high frequency shaking with frequencies > 10 kHz.

A complete new feature in shock generation is the µs precise timing capability, what allows synchronizing of mechanical shock events with other fast physical processes.
Fixed "Phased Arrays" of piezo-stack based shock generators will allow shock front generation with variable propagation characteristics.

Piezo-stacks are robust devices, which allow the permanent integration into extended mechanical structures like buildings etc. for long term structure health monitoring.
In an inverted operation mode, the described piezo-system can be used as shock absorber.

Shock wave theory in bars and rods (Impact in rods)

9

9.1 Impact propagation in the ideal rod

The ideal rod is characterized by a small diameter to length ratio. An elastic signal is propagated with the related speed of sound without changing the form of the signal according to the ansatz of d'Alembert to solve the wave equation (s. Timoshenko). To be exact this stress impulse should not be described as wave or shock impulse, but as these designations are already very common in literature we used it in the first part of this paper. Here in the theoretical section we shall use stress impulse instead of shock wave.

For the axial propagation of elastic stress impulses the theory of thin rods (Graff, Johnson) can be applied. A stress signal launched from the left in a given homogenous rod with cross section a will be propagated to the right with the speed of sound (Figure 12) c given by

$$c = \sqrt{(E/\rho)}$$

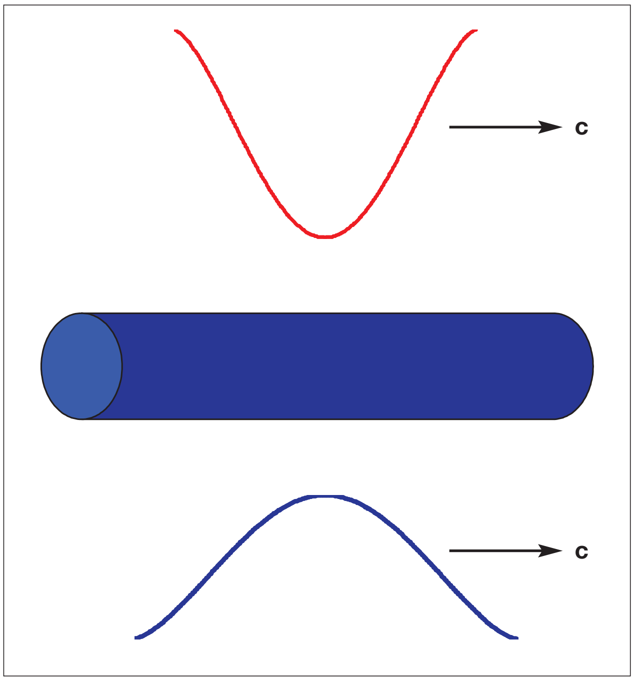

Figure 12: *Stress impulse (top) and related particle velocity (bottom).*

Synchronously with the stress impulse sigma (t) the signal of the particle velocity v(t) propagates along the rod (Figure 12) where the fundamental relation holds

$$\sigma(t) = -I\ v(t)$$

A consequence of the 2. law of Newton. I represents the acoustical impedance given by

$$I = \rho\ c$$

The law of Hooke correlates stress σ and strain ε according to

$$\sigma = E\varepsilon$$

where E is Young's modulus.

A fundamental property of a stress impulse is the equality of elastic and kinetic energy during propagation, except at the ends of the rod and as long as cross section and/or impedance do not change. A harmonic wave behaves different, it periodically exchanges elastic and kinetic energy completely with the frequency of the wave.

In general two stress impulses propagate independently in both directions. The resulting stress is the sum and given as a function of position and time

$$\sigma\ (x,t) = \sigma_{right}\ (x\text{-}c\ t,t) + \sigma_{left}\ (x+c\ t,t).$$

For steel c is about 5200 m/s.

The corresponding particle velocity as a function of position and time reads

$$v(x,t) = 1/I\ (\sigma_{right}\ (x,t) - \sigma_{left}\ (x,t))$$

with the designation

stress	σ
particle velocity	v
impulse momentum	P
energy	W
impedance	I

The impedance of a steel rod is about 4 10^7 kg/(m^2s) and for a stress impulse of 200 MPa the particle velocity is about 5 m/s. The impulse energy as mentioned above is to one half elastic which is given by the integral

$$W = \frac{a}{2 \cdot I} \int_0^T \sigma(t)^2 dt$$

or the kinetic part

$$W = \frac{a \cdot I}{2} \cdot \int_0^T v(t)^2 dt$$

The techniques to measure these two energy contributions are different. For the elastic part strain gages are used, for the kinetic energies Laser Doppler Anemometry (LDA) is applied.

The propagated impulse momentum $P_{Impulse}$ is determined by the integral

$$P_{Impuls} = a \cdot \int_0^{T_{Impuls}} \acute{o}(t) \cdot dt$$

The impulse reaching the end of the rod will be reflected. If the end is acoustically free (soft) the stress amplitude changes sign, the particle velocity not. If the end is acoustically hard (fixed), it is vica versa. At the end of the rod all energy is in the first case kinetic and in the second case completely elastic.

For the change of the rod cross section areas from a_1 to a_2.

Simple transmission - τ and reflection - r rules are valid for the stress:

$$\tau = 2\, a_1/(a_1 + a_2)$$
$$r = (a_2 - a_1)/(a_1 + a_2)$$

Equivalent rules are given, if the impedances I_1, I_2 change

$$\tau = 2\, I_2/(I_1 + I_2)$$
$$r = (I_2 - I_1)/(I_1 + I_2)$$

or if both, areas and impedances change

$$\tau = 2\, a_1 I_2/(a_1 I_1 + a_2 I_2)$$
$$r = (a_2 I_2 - a_1 I_1)/(a_1 I_1 + a_2 I_2)$$

The reflection will be zero if the relation $I_2/I_1 = a_1/a_2$ holds.

For the particle velocities similar rules are valid, not given here (Johnson).

The limit of the simple rod theory is reached if the lateral dimension of the rod has to be taken into account leading to additional inertia effects. The stress state is no longer constant along the cross section (no plane situation).

9.2 Stress impulse generation by a ramming rod

Usually a stress impulse can be generated by the impact of two cylindrical rods. An example are the impacts of flying piston, ram and shaft of the Hilti EP-tool (Electropneumatic Percussion). Here now the fundamental difference between the real hammer and the piezo-hammer is based: a hammer is ramming - the piezo-actuator is impacting.

If the surfaces of contact are spherical the contact theory of Hertz (Johnson) can be applied. The contact of ideal plane surfaces has to be treated acoustically or as the numerical limit of very large contact radii.

Definitions:
The indices 1 and 2 are related to the rod 1 and rod 2, respectively.
E is the Young's modulus, ν is Poisson's ratio. For the effective modulus E_{eff}:

$$E_{eff} = (1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2$$

holds. For the effective contact radius r_{eff} the equation hold:

$$1/r_{eff} = 1/r_1 + 1/r_2$$

With E_{eff} und r_{eff} the stiffness constant of the Hertzian contact k_{Hertz} is written

$$k_{Hertz} = 4/3\ E_{eff}\ r_{eff}^{1/2}$$

The force of contact is given by

$$f_{contact}\ (w) = k_{Hertz} w^{3/2}$$

where w is the interpenetration of the two surfaces.

Dynamically the Hertzian contact between rod 1 und rod 2 with the effective mass m_{eff}

$$1/m_{eff} = 1/m_1 + 1/m_2)$$

and the dynamic force

$$f_{contact}(w(t)) = k_{Hertz} w(t)^{3/2}$$

lasts during the contact time $T_{contact}$

$$T_{contact} = 2.9432\ (15/16)^{2/5}\ (1/E_{eff})^{2/5}\ (m_{eff})^{2/5}\ (1/_{reff})^{1/5}\ (1/v)^{1/5}$$

with v beeing the relative velocity of the two rods before impact.

Before impact, the rod (e.g. the flying piston) is flying freely and can be represented in terms of two stress impulses one to the right σ_{right} and a second to the left σ_{left} both with the same amount σ_0

$$\sigma_0 = I v_0/2$$

but with opposite sign. For the particle velocity the relation holds

$$1/I(\sigma_{rechts} - \sigma_{links}) = v_0$$

During the contact energy and impulse momentum are exchanged.

For a dynamic Hertzian contact the force of contact is in good approximation given by an Gaussian signal form where impulse maximum and impulse width depend on the given parameters. In the example of figure 13 the maximum is assumed to 50 kN and the width to 50 µs.

9.2 Stress impulse generation by a ramming rod

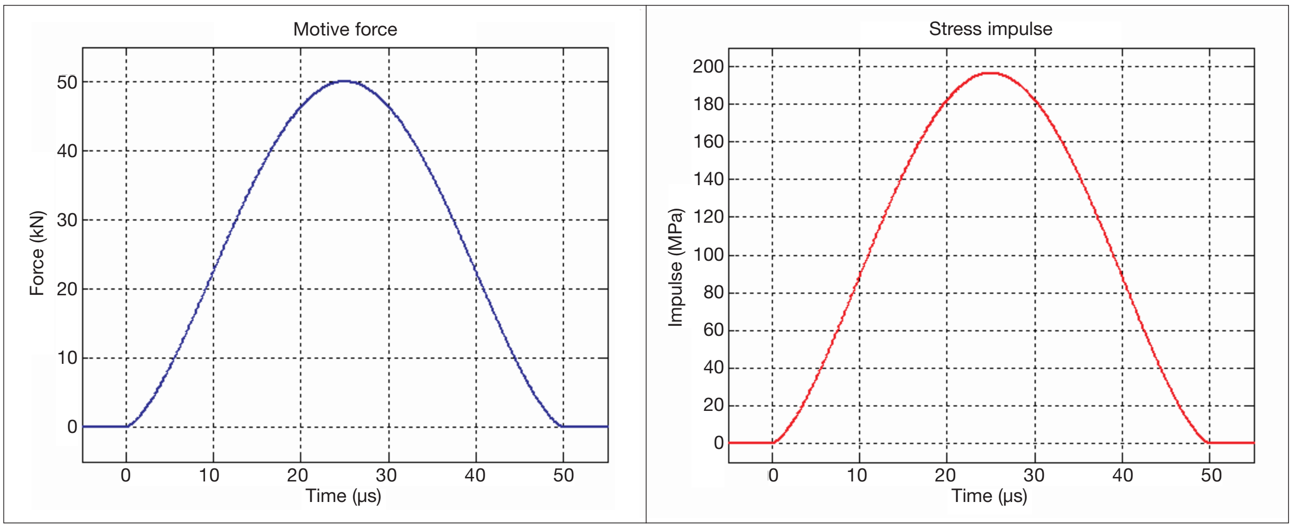

Figure 13: *Force of contact as a function of time (left) and stress impulse as a function of time (right).*

The maximum force of 50 kN (5t) in the rod of 18 mm diameter corresponds to a stress of about 200 MPa (the tension stress of Steel St52 is about 520 MPa).

Force and stress are related by

$$\sigma(t) = \frac{f(t)}{a}$$

a is the cross section of the rod. The change of impulse momentum Δp:

$$\Delta p = \int_0^T f(t)dt$$

is the integral over the force as a function of time (second law of Newton). $\Delta p = 1.3$ kgm/s.

The Gaussian signal form can be represented in good approximation by

$$\sigma = \sigma_0 \sin(\pi t/T)^{3/2}.$$

and the energy can then be analytically calculated to

$$W = 4/3\ T/\pi\ a/l\ \sigma_0^2$$

It is worthwhile to mention that impact and a single harmonic wave do not correspond. The Gaussian like stress impulse needs a Gaussian Fourier distribution of frequencies in time and space.
A stress impulse running for- and backward within the rod has only the period in common with the fundamental resonance vibration.

9.3 Piezo-actuator as active rod

There are two piezo-effects the direct and the inverse (both 1880 invented by J. und P. Curie at cristalls of quartz).

The direct piezo-effect will not be treated here. The inverse piezo-effect is described by the state equation in scalar form

$$\varepsilon = c^E\sigma + d_{33}\,E$$

ε = strain
E = electrical field
c^E = elastic constant of compliance at constant electrical field (e. g. E = 0).
σ = stress
d_{33} is the axial piezo-electrical constant

For $\sigma = 0$ the strain is direct proportional to the electrical field. If the strain is blocked ($\varepsilon = 0$), then

$$\sigma = -d_{33}/c^E\,E$$

representing a compression. This relation is the basis of the stress impulse generation using piezo-actuators. As the piezo-actuator is a cylindrical rod we call this special type of rod active.

Connecting all piezo-disks of the stack at an instant of time with an electrical voltage (see chapter 5) the piezo-rod (stack) will react with an instant strain. The form of an active rod can be represented by two stress impulses σ_{right} und σ_{left} with equal amount σ_0 and equal sign but opposite propagation directions. I.e. the particle velocities are also equal in amount but opposite in propagation direction. They are compensating at the beginning, contrary to the free flying rod where the stresses are compensating.

For the stress energy follows

$$W = ½\,V_{piezo}\,E_{piezo}\,d_{33}^2\,(U_0/d)^2$$

V_{piezo} volumen of the piezo-stack
E_{piezo} Young's modulus of the stack
U_0/d electrical field strength given by the ratio of voltage U_0 and d
d thickness of an individual piezo-disk

In reality the stress impulse is not ideal square. Due to finite rise time of the electrical voltage signal convolution effects are rounding the ideal square stress impulse form.

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